

Systematics of string loop corrections in type IIB Calabi-Yau flux compactifications

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ABSTRACT: We study the behaviour of the string loop corrections to the $N = 1$ 4D supergravity Kähler potential that occur in flux compactifications of IIB string theory on general Calabi-Yau three-folds. We give a low energy interpretation for the conjecture of Berg, Haack and Pajer for the form of the loop corrections to the Kähler potential. We check the consistency of this interpretation in several examples. We show that for arbitrary Calabi-Yaus, the leading contribution of these corrections to the scalar potential is always vanishing, giving an “extended no-scale structure”. This result holds as long as the corrections are homogeneous functions of degree -2 in the 2-cycle volumes. We use the Coleman-Weinberg potential to motivate this cancellation from the viewpoint of low-energy field theory. Finally we give a simple formula for the 1-loop correction to the scalar potential in terms of the tree-level Kähler metric and the conjectured correction to the Kähler potential. We illustrate our ideas with several examples. A companion paper will use these results in the study of Kähler moduli stabilisation.

KEYWORDS: Superstring Vacua, Supersymmetric Effective Theories, Flux compactifications.

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1. Introduction

Four-dimensional effective actions have played a major rôle in addressing the moduli stabilisation problem of string compactifications (for recent reviews with many references see [1, 2]). Most of these efforts rely on Calabi-Yau backgrounds with fluxes of RR fields for which a worldsheet understanding of string interactions is not available and the effective action approach is the only reliable tool at present to study the moduli dynamics. For moduli stabilisation and the study of other low-energy and cosmological implications, it is then important to have control on the $\mathcal{N} = 1$ supergravity effective actions associated to string compactifications.

The Kähler potential K is the least understood part of these four-dimensional effective actions. There has been substantial progress in determining the tree-level structure of the

Kähler potential as a function of the many moduli fields appearing in arbitrary Calabi-Yau compactifications [3]. However, unlike the superpotential W , the lack of holomorphy implies that the Kähler potential can receive corrections to all orders in the α' and g_s expansions. The presence of no-scale structure in the Kähler potential makes understanding the Kähler corrections particularly pressing, as it is the corrections that give rise to the leading perturbative terms in the scalar potential.

Mirror symmetry and the underlying $\mathcal{N} = 2$ structure was used to extract the leading order α' corrections [4]. Explicit string amplitude calculations to determine the loop corrections to K are not available for general fluxed Calabi-Yau compactifications, and only simple unfluxed toroidal orientifold cases have been used for concrete computations [5].

Despite the difficulty of explicitly computing loop corrections in general Calabi-Yau flux backgrounds, given their importance it is necessary to try and go as far as possible. In this respect we observe that there is an easier and a harder part to computing the form of loop corrections. The easier part involves the parametric scaling of moduli that control the loop expansion - in IIB these are the dilaton, which controls the string coupling, and the Kähler moduli, which controls the gauge coupling on D7 branes. The harder part involves the actual coefficients of the loop expansion, which depend on the complex structure moduli and would require an explicit string computation. This article focuses entirely on the ‘easier’ part; however as the Kähler moduli are unstabilised at tree-level, such knowledge is very important for moduli stabilisation.

Recently, Berg, Haack and Pajer (BHP) [6] gave arguments for the general functional dependence of the leading order loop corrections to K on the Kähler moduli. By comparing with the toroidal orientifold calculations and the standard transformations required to go from the string frame, where string amplitudes are computed, to the physical Einstein frame that enters the supergravity action, they conjectured the parametric form of the leading corrections for general Calabi-Yau compactifications as a function of the Kähler moduli. As mentioned above, it is this dependence (on the Kähler moduli) that is more relevant for moduli stabilisation, as the dilaton and complex structure moduli are usually stabilised directly from the fluxes and it is only the Kähler moduli that need quantum corrections to the scalar potential to be stabilised. These quantum corrections include non-perturbative corrections to the superpotential W (since W is not renormalised perturbatively [7]), α' and string loop corrections to K . Non-perturbative corrections to K are subdominant with respect to the perturbative corrections. It is then of prime importance to have control on the parametric form of the quantum corrections to K .

In this article we study in detail the leading order loop corrections to K conjectured by BHP. We provide a low-energy interpretation of this conjecture and give a general argument that, with the conjectured form, the leading loop corrections to the Kähler potential cancel at leading order in their contributions to the scalar potential. This is very relevant for the robustness of the large volume scenario [8] for which the leading α' corrections were used to obtain stabilised exponentially large volumes. Even though the leading string loop correction to K is dominant over the α' corrections, its contribution to the scalar potential is subdominant [5, 6]. We will also see that this cancellation is necessary if the loop corrections to K are to generate corrections to the scalar potential consistent with those

expected from the Coleman-Weinberg potential. We also extend this result to more general possible corrections to K , showing that the only property needed for the cancellation is that δK is a homogeneous function of degree $n = -2$, which includes the BHP proposal. We illustrate our results with several examples.

This article is organized as follows. In section 2 we review the present status of the tree-level and quantum corrected effective actions and their rôle for moduli stabilisation. Sections 3 and 4 are the main parts of the article in which we study in detail the proposed form of the string loop corrections to the Kähler potential, their interpretation in terms of the Coleman-Weinberg potential and examples of different Calabi-Yau manifolds where these corrections are relevant. Finally in a comprehensive appendix A we provide a general discussion of the different proposals that have been put forward to stabilise Kähler moduli, emphasizing that in all cases it is necessary to understand the quantum corrections to K . This is an explicit illustration of the importance to better understand the perturbative corrections to the supersymmetric action. In particular the ‘extended no-scale structure’ of section 4 is crucial to establish the robustness of the exponentially large volume scenario. In a companion article [9] we will use our results to study moduli stabilisation in different classes of Calabi-Yau manifolds.

2. Effective action for type IIB flux compactifications

2.1 Tree-level action

We first review the low energy theory of IIB Flux Compactifications on a Calabi-Yau X [10]. The tree-level superpotential is generated by turning on fluxes and takes the Gukov-Vafa-Witten form:

$$W_{\text{tree}}(S, U) = \int_X G_3 \wedge \Omega, \tag{2.1}$$

with $G_3 = F_3 + iSH_3$, where F and H are RR and NSNS 3-form fluxes respectively, S is the axio-dilaton $S = e^{-\varphi} + iC_0$, (with e^φ the string coupling and C_0 the RR 0-form), and Ω is the holomorphic (3,0)-form which depends on the complex structure moduli U . The tree level Kähler potential K_{tree} is

$$K_{\text{tree}} = -2 \ln(\mathcal{V}) - \ln(S + \bar{S}) - \ln \left(-i \int_X \Omega \wedge \bar{\Omega} \right), \tag{2.2}$$

where \mathcal{V} is the Einstein frame internal volume that depends only on $(T + \bar{T})$. K_{tree} has a factorized form with respect to T , U and S moduli. The T moduli are defined by

$$T_i = \tau_i + ib_i, \tag{2.3}$$

where τ_i is the Einstein frame volume of a 4-cycle Σ_i , measured in units of $l_s = (2\pi)\sqrt{\alpha'}$, and b_i is the component of the RR 4-form C_4 along this cycle: $\int_{\Sigma_i} C_4 = b_i$. The 4-cycle volumes τ_i may be related to the 2-cycle volumes t_i . Letting D_i be a basis of divisors on

X (we use D_i to denote both the divisor and its dual 2-form), and k_{ijk} the divisor triple intersections, the overall volume \mathcal{V} can be written as

$$\mathcal{V} = \frac{1}{6} \int_X J \wedge J \wedge J = \frac{1}{6} k_{ijk} t^i t^j t^k, \quad (2.4)$$

where $J = t^i D_i$ is the Kähler form. The 4-cycle volumes τ_i are defined as

$$\tau_i = \frac{\partial \mathcal{V}}{\partial t^i} = \frac{1}{2} \int_X D_i \wedge J \wedge J = \frac{1}{2} k_{ijk} t^j t^k. \quad (2.5)$$

Finally, let us introduce the following notation

$$A_{ij} = \frac{\partial \tau_i}{\partial t^j} = \int_X D_i \wedge D_j \wedge J = k_{ijk} t^k. \quad (2.6)$$

Some useful relations that we will use subsequently are

$$\begin{cases} t^i \tau_i = 3\mathcal{V}, \\ A_{ij} t^j = 2\tau_i, \\ A_{ij} t^i t^j = 6\mathcal{V}, \end{cases} \quad (2.7)$$

along with

$$K_i^0 \equiv \frac{\partial (K_0)}{\partial \tau_i} = -\frac{t_i}{\mathcal{V}}, \quad (2.8)$$

where $K_0 = -2 \ln(\mathcal{V})$. In addition, the general form of the Kähler matrix is

$$K_{ij}^0 \equiv \frac{\partial^2 (K_0)}{\partial \tau_i \partial \tau_j} = \frac{1}{2} \frac{t_i t_j}{\mathcal{V}^2} - \frac{A^{ij}}{\mathcal{V}}, \quad (2.9)$$

and its inverse looks like

$$K_0^{ij} \equiv \left(\frac{\partial^2 (K_0)}{\partial \tau_i \partial \tau_j} \right)^{-1} = \tau_i \tau_j - \mathcal{V} A_{ij}. \quad (2.10)$$

For later convenience, we have expressed the derivatives of the Kähler potential in terms of derivatives with respect to $\tau = \text{Re}(T)$ rather than derivatives with respect to T (this accounts for some differences in factors of 2 in certain equations compared to the literature). From the previous relations it is also possible to show that

$$K_0^{ij} K_i^0 = -\tau_j, \quad (2.11)$$

and the more important result

$$K_0^{ij} K_i^0 K_j^0 = 3. \quad (2.12)$$

The $\mathcal{N} = 1$ F-term supergravity scalar potential is given by:

$$V = e^K \left\{ K^{SS} D_S W D_S \bar{W} + K^{UU} D_U W D_U \bar{W} + 4K^{ij} D_i W D_j \bar{W} - 3|W|^2 \right\}, \quad (2.13)$$

where

$$\begin{cases} D_i W = \frac{\partial W}{\partial \tau_i} + \frac{1}{2} W \frac{\partial K}{\partial \tau_i} \equiv W_i + \frac{1}{2} K_i W, \\ D_j \bar{W} = \frac{\partial \bar{W}}{\partial \tau_j} + \frac{1}{2} \bar{W} \frac{\partial K}{\partial \tau_j} \equiv \bar{W}_j + \frac{1}{2} K_j \bar{W}. \end{cases} \quad (2.14)$$

The form of the scalar potential given in (2.13) has used the factorization of the moduli space: in general this will be lifted by quantum corrections. As W_{tree} is independent of the Kähler moduli, this reduces to

$$V = e^K \left\{ K^{SS} D_S W D_S \bar{W} + K^{UU} D_U W D_U \bar{W} + (K^{ij} K_i K_j - 3) |W|^2 \right\}. \quad (2.15)$$

Furthermore, (2.12) implies the existence of no scale structure as the last term of (2.15) vanishes:

$$V = e^K \left\{ K^{SS} D_S W D_S \bar{W} + K^{UU} D_U W D_U \bar{W} \right\} \geq 0. \quad (2.16)$$

As the scalar potential is positive semi-definite it is possible to fix the dilaton and the complex structure moduli by demanding $D_S W = 0 = D_U W$. Usually, these fields are integrated out setting them equal to their vacuum expectation values but sometimes we will keep their dependence manifest. However since they are stabilised at tree level, even though they will couple to quantum corrections, these will only lead to subleading corrections to their VEVs, so it is safe just to integrate them out. From now on, we will set

$$W_0 = \left\langle \int_X G_3 \wedge \Omega \right\rangle. \quad (2.17)$$

2.2 Non-perturbative and α' corrections

As seen in the previous paragraph, at tree level we can stabilise only the dilaton and the complex structure moduli but not the Kähler moduli. The only possibility to get mass for these scalar fields is thus through quantum corrections.

It is known that in $N=1$ 4D *SUGRA*, the Kähler potential receives corrections at every order in perturbations theory, while the superpotential receives non-perturbative corrections only, due to the non-renormalisation theorem. The corrections will therefore take the general form:

$$\begin{cases} K = K_{\text{tree}} + K_p + K_{np}, \\ W = W_{\text{tree}} + W_{np}, \end{cases} \quad (2.18)$$

and the hope is to stabilise the Kähler moduli through these quantum corrections. In this section we will review the behaviour of the non-perturbative and α' corrections and then study the g_s corrections in the main part of our paper.

Non-perturbative corrections to the superpotential are given by an infinite series of contributions

$$W_{np} = \sum_{i,m} A_{i,m}(S, U) e^{-m a_i T_i}. \quad (2.19)$$

They can arise from either Euclidean D3-brane instantons ($a_i = 2\pi$) or gaugino condensation on wrapped D7-branes ($a_i = 2\pi/N$, with N the rank of the condensing gauge group). In general, $A_{i,m}$ depend on both dilaton and complex structure moduli. We will always

work in a regime where $a_i \tau_i \gg 1 \forall i = 1, \dots, h_{1,1}$ so that we can ignore higher instanton corrections and keep just the leading non-perturbative corrections:

$$W_{np} = \sum_i A_i(S, U) e^{-a_i T_i}. \tag{2.20}$$

K_{np} can come from either worldsheet or brane instantons and is subdominant compared to the perturbative corrections to the Kähler potential (see for instance [11, 12]) which in general come from both the α' and the g_s expansion

$$K_p = \delta K_{(\alpha')} + \delta K_{(g_s)}. \tag{2.21}$$

The leading α' correction to the Kähler potential comes from the ten dimensional $\mathcal{O}(\alpha'^3) \mathcal{R}^4$ term. It has been computed in [4] and reads

$$\begin{aligned} K_0 + \delta K_{(\alpha')} &= -2 \ln \left(\mathcal{V} + \frac{\xi}{2} \text{Re}(S)^{3/2} \right) = \\ &= -2 \ln(\mathcal{V}) - \frac{\xi \text{Re}(S)^{3/2}}{\mathcal{V}} + \mathcal{O}(1/\mathcal{V}^2), \end{aligned} \tag{2.22}$$

where the constant ξ is given by

$$\xi = -\frac{\chi(X)\zeta(3)}{(2\pi)^3}, \tag{2.23}$$

with $\chi(X) = 2(h^{1,1} - h^{2,1})$ and $\zeta(3) \equiv \sum_{k=1}^{\infty} 1/k^3 \simeq 1.2$. We stress the point that the α' expansion is an expansion in inverse volume and thus can be controlled only at large volume. This is important, as very little is known about higher α' corrections, the exact form of which are not known even in the maximally supersymmetric flat 10D IIB theory. From now on we focus only on situations in which the volume can be stabilised at $\mathcal{V} \gg 1$ in order to have theoretical control over the perturbative expansion in the low-energy effective field theory. The inclusion of (2.20) and (2.22) now gives the following scalar potential (where the dilaton has been fixed and the factor $\text{Re}(S)^{3/2}$ included in the definition of ξ)

$$\begin{aligned} V &= V_{np} + V_{(\alpha')} \\ &= e^K \left[K^{jk} \left(a_j A_j a_k \bar{A}_k e^{-(a_j T_j + a_k \bar{T}_k)} - \left(a_j A_j e^{-a_j T_j} \bar{W} K_k + a_k \bar{A}_k e^{-a_k \bar{T}_k} W K_j \right) \right) \right. \\ &\quad \left. + 3\xi \frac{(\xi^2 + 7\xi\mathcal{V} + \mathcal{V}^2)}{(\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} |W|^2 \right]. \end{aligned} \tag{2.24}$$

3. General analysis of the string loop corrections

3.1 String loop corrections

Our discussion of the form of the scalar potential in IIB flux compactifications has still to include the string loop corrections $\delta K_{(g_s)}$. These have been computed in full detail only for unfluxed toroidal orientifolds in [5]. Subsequently the same collaboration in [6] made an

educated guess for the behaviour of these loop corrections for general smooth Calabi-Yau three-folds by trying to understand how the toroidal calculation would generalize to the Calabi-Yau case. To be self-contained, we therefore briefly review the main aspects of the toroidal orientifold calculation of [5].

3.1.1 Exact calculation: $N=2$ $K3 \times T^2$ and $N=1$ $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

The string loop corrections to $N=1$ supersymmetric $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold compactifications with D3 and D7 branes follow by generalising the result for $N=2$ supersymmetric $K3 \times T^2$ orientifolds. Therefore we start by outlining the result in the second case.

The one-loop corrections to the Kähler potential from Klein bottle, annulus and Möbius strip diagrams are derived by integrating the one-loop correction to the tree level Kähler metric. These corrections are given by 2-point functions and to derive the corrections $\delta K_{(g_s)}$ it is sufficient to compute just one of these correlators and integrate, since all corrections to the Kähler metric come from the same $\delta K_{(g_s)}$. From [5] the one-loop correction to the 2-point function of the complex structure modulus U of T^2 is given by, dropping numerical factors,

$$\langle V_U V_{\bar{U}} \rangle \sim - (p_1 \cdot p_2) g_s^2 \alpha'^{-4} V_4 \frac{vol(T^2)_s}{(U + \bar{U})^2} \mathcal{E}_2(A_i, U), \tag{3.1}$$

where V_4 is the regulated volume of the $4D$ spacetime, $vol(T^2)_s$ denotes the volume of T^2 in string frame and A_i are open string moduli. The coefficient $\mathcal{E}_2(A_i, U)$ is a linear combination of non-holomorphic Eisenstein series $E_2(A, U)$ given by

$$E_2(A, U) = \sum_{(n,m) \neq (0,0)} \frac{Re(U)^2}{|n + mU|^4} \exp \left[2\pi i \frac{A(n + m\bar{U}) + \bar{A}(n + mU)}{U + \bar{U}} \right]. \tag{3.2}$$

The result (3.1) is converted to Einstein frame through a Weyl rescaling

$$\langle V_U V_{\bar{U}} \rangle_E = \langle V_U V_{\bar{U}} \rangle_s \frac{e^{2\varphi}}{vol(K3 \times T^2)_s}, \tag{3.3}$$

giving

$$\langle V_U V_{\bar{U}} \rangle \sim - (p_1 \cdot p_2) g_s^2 \alpha'^{-4} V_4 \frac{e^{2\varphi}}{(U + \bar{U})^2} \frac{\mathcal{E}_2(A_i, U)}{vol(K3)_s}. \tag{3.4}$$

Writing the volume of the $K3$ hypersurface in Einstein frame $vol(K3)_s = e^\varphi vol(K3)_E$, produces the final result

$$\langle V_U V_{\bar{U}} \rangle \sim - (p_1 \cdot p_2) g_s^2 \alpha'^{-4} V_4 \frac{e^\varphi}{(U + \bar{U})^2} \frac{\mathcal{E}_2(A_i, U)}{vol(K3)_E}. \tag{3.5}$$

Now noticing that

$$\partial_U \partial_{\bar{U}} E_2(A, U) \sim - \frac{E_2(A, U)}{(U + \bar{U})^2}, \tag{3.6}$$

we can read off from (3.5) the 1-loop correction to the kinetic term for the field U and using $vol(K3)_E = \tau$, the 1-loop correction to the Kähler potential becomes

$$\delta K_{(g_s)} = c \frac{\mathcal{E}_2(A_i, U)}{Re(S) \tau}, \tag{3.7}$$

where a full analysis determines the constant of proportionality c to be $c = -1/(128\pi^4)$.¹ This procedure can be generalized to evaluate the loop corrections in the $N=1$ supersymmetric $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ case, obtaining

$$\delta K_{(g_s)} = \delta K_{(g_s)}^{KK} + \delta K_{(g_s)}^W, \tag{3.8}$$

where $\delta K_{(g_s)}^{KK}$ comes from the exchange between D7 and D3-branes of closed strings which carry Kaluza-Klein momentum, and gives (for vanishing open string scalars)

$$\delta K_{(g_s)}^{KK} = -\frac{1}{128\pi^4} \sum_{i=1}^3 \frac{\mathcal{E}_i^{KK}(U, \bar{U})}{\text{Re}(S) \tau_i}. \tag{3.9}$$

The other correction $\delta K_{(g_s)}^W$ can again be interpreted in the closed string channel as coming from the exchange of winding strings between intersecting stacks of D7-branes. These contributions are present in the $N=1$ case but not in the $N=2$ case. They take the form

$$\delta K_{(g_s)}^W = -\frac{1}{128\pi^4} \sum_{i \neq j \neq k=1}^3 \frac{\mathcal{E}_i^W(U, \bar{U})}{\tau_j \tau_k}. \tag{3.10}$$

3.1.2 Generalisation to Calabi-Yau three-folds

The previous calculation teaches us that, regardless of the particular background under consideration, a Weyl rescaling will always be necessary to convert to four-dimensional Einstein frame. This implies the 2-point function should always be suppressed by the overall volume:

$$\langle V_U V_{\bar{U}} \rangle_s \sim g(U, T, S) \iff \langle V_U V_{\bar{U}} \rangle_E \sim g(U, T, S) \frac{e^{\varphi/2}}{\mathcal{V}_E}. \tag{3.11}$$

This allowed [6] to conjecture the parametric form of the loop corrections even for Calabi-Yau cases. $g(U, T, S)$ originates from KK modes as m_{KK}^{-2} and so should scale as a 2-cycle volume t . Conversion to Einstein frame then leads to

$$\delta K_{(g_s)}^{KK} \sim \sum_{i=1}^{h_{1,1}} g(U) \frac{(a_i t^l) e^\varphi}{\mathcal{V}} = \sum_{i=1}^{h_{1,1}} \frac{\mathcal{C}_i^{KK}(U, \bar{U}) (a_i t^l)}{\text{Re}(S) \mathcal{V}}, \tag{3.12}$$

where $a_i t^l$ is a linear combination of the basis 2-cycle volumes t_l . A similar line of argument for the winding corrections (where the function $g(U, T, S)$ goes as $m_W^{-2} \sim t^{-1}$) gives

$$\delta K_{(g_s)}^W \sim \sum_{i=1}^{h_{1,1}} \frac{\mathcal{C}_i^W(U, \bar{U})}{(a_i t^l) \mathcal{V}}. \tag{3.13}$$

Notice that \mathcal{C}_i^{KK} and \mathcal{C}_i^W are unknown functions of the complex structure moduli and therefore this mechanism is only useful to fix the leading order dependence on Kähler

¹The constant c given here differs from the one calculated in [5] only by a factor of $(-\pi^2)$ due to different conventions. In fact, in [5] the correction (3.7) takes the form $\delta K_{(g_s)} = -\frac{c}{8} \frac{\mathcal{E}_2(A_i, U)}{\text{Im}(S) \text{Im}(T)}$ with $\text{Im}(S) \equiv \frac{e^{-\varphi}}{\sqrt{8\pi}}$ and $\text{Im}(T) \equiv \frac{\tau}{\sqrt{8\pi}}$.

moduli. This is similar to the Kähler potential for matter fields whose dependence on Kähler moduli can be extracted by scaling arguments [13], while the complex structure dependence is unknown. Fortunately it is the Kähler moduli dependence that is more relevant in both cases due to the fact that complex structure moduli are naturally fixed by fluxes at tree-level. On the other hand, the Kähler moduli need quantum corrections to be stabilised and are usually more relevant for supersymmetry breaking.

We now turn to trying to understand the loop corrections from a low-energy point of view.

3.2 Low energy approach

The low energy physics is described by a four dimensional supergravity action. We ask here whether it is possible to understand the form of the loop corrections in terms of the properties of the low energy theory, without relying on a full string theory computation.

We first ask what one could reasonably hope to understand. The form of equations (3.9) and (3.10) show a very complicated dependence on the complex structure moduli, and a very simple dependence on the dilaton and Kähler moduli. The dependence on the complex structure moduli is associated with an Eisenstein series originating from the structure of the torus, and so we cannot expect to reproduce this without a full string computation. On the other hand the dilaton and Kähler moduli appear with a very simple scaling behaviour. This we may hope to be able to understand using low-energy arguments, and to be able to conjecture the generalisation to the Calabi-Yau case.

There is one paper in the literature that has already tried to do that. In an interesting article [14], von Gersdorff and Hebecker considered models with one Kähler modulus τ , such that $\mathcal{V} = \tau^{3/2} = R^6 \iff \tau = R^4$, and argued for the form of $\delta K_{(g_s)}^{KK}$ using the Peccei-Quinn symmetry, scaling arguments and the assumption that the loop corrections arise simply from the propagation of 10D free fields in the compact space and therefore do not depend on M_s . This led to the proposal

$$\delta K_{(g_s)}^{KK} \simeq \tau^{-2}. \tag{3.14}$$

However, at the level of the Kähler potential (but not the scalar potential) this result disagrees with the outcome of the exact toroidal calculation (3.9). It seems on the contrary to reproduce the corrections due to the exchange of winding strings (3.10), but as $m_W > M_s > m_{KK}$ we do not expect to see such corrections at low energy. In reality, $\delta K_{(g_s)}^{KK}$ should contain all contributions to the 1-loop corrections to the kinetic term of τ . From the reduction of the DBI action we know that τ couples to the field theory on the stack of $D7$ -branes wrapping the 4-cycle whose volume is given by τ . It therefore does not seem that the string loop corrections will come from the propagation of free fields as τ will interact with the corresponding gauge theory on the brane. In fact the reduced DBI action contains a term which looks like

$$\delta S_{\text{DBI}} \supset \int d^4x \sqrt{-g^{(4)}} \tau F^{\mu\nu} F_{\mu\nu}, \tag{3.15}$$

and when τ gets a non-vanishing VEV, expanding around this VEV in the following way

$$\tau = \langle \tau \rangle + \tau', \tag{3.16}$$

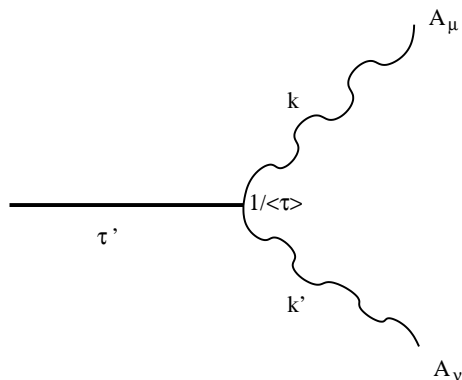


Figure 1: Coupling of the Kähler modulus with the gauge fields on the brane.

we obtain

$$\delta S_{\text{DBI}} \supset \int d^4x \sqrt{-g^{(4)}} (\langle \tau \rangle F^{\mu\nu} F_{\mu\nu} + \tau' F^{\mu\nu} F_{\mu\nu}). \quad (3.17)$$

From the first term in (3.17) we can readily read off the coupling constant of the gauge group on the brane

$$g^2 = \frac{1}{M_s^4 \tau}, \quad (3.18)$$

where we have added M_s^4 to render it correctly dimensionless. On the other hand, the second term in (3.17) will give rise to an interaction vertex of the type shown in figure 1 that will affect the 1-loop renormalisation of the τ kinetic term.

In any ordinary quantum field theory, generic scalar fields φ get 1-loop quantum corrections to their kinetic terms (wavefunction renormalisation) of the form

$$\int d^4x \sqrt{-g^{(4)}} \frac{1}{2} (1 + A) \partial_\mu \varphi \partial^\mu \varphi, \quad (3.19)$$

where A is given by $A \simeq \frac{g^2}{16\pi^2}$, with g the coupling constant of the gauge interaction this scalar couples to.

τ is a modulus and not a gauge-charged field. Nonetheless, we still expect loop corrections to generate corrections to the moduli kinetic terms. We expect to be able to write the kinetic terms as

$$K_{i\bar{j}} = K_{i\bar{j},tree} + \delta K_{i\bar{j},1-loop}. \quad (3.20)$$

We also expect the loop correction to the kinetic term to always involve a suppression by the coupling that controls the loop expansion. This is the analogue to the correction in (3.19) depending on the gauge coupling constant, which controls the loop expansion of ordinary field theory. For a brane wrapping a cycle τ , the value of τ is the gauge coupling for branes wrapping the cycle, and we expect loop corrections involving those branes to involve a suppression, relative to tree-level terms, by a factor of τ (see [15] for related arguments).

This is not a rigorous derivation, but we consider this a reasonable assumption. We will find that it gives the correct scaling of the loop correction for the toroidal case where the

correction has been computed explicitly, and that, while it has a different origin, it agrees with the BHP conjecture for the parametric form of loop corrections in the Calabi-Yau case. The loop corrections to the Kähler potential K should then be such as to generate corrections to the kinetic terms for τ that are suppressed by a factor of g^2 for the gauge theory on branes wrapping the cycle τ . The Kähler potential upon double differentiation yields the kinetic terms in the $4D$ Einstein frame Lagrangian

$$S_{Einstein} \supset \int d^4x \sqrt{-g^{(4)}} \left(\frac{\partial^2 (K_{\text{tree}})}{\partial \tau^2} + \frac{\partial^2 (\delta K_{(g_s)}^{KK})}{\partial \tau^2} \right) (\partial \tau)^2, \quad (3.21)$$

and the general canonical redefinition of the scalar fields

$$\tau \longrightarrow \varphi = \varphi(\tau), \quad (3.22)$$

will produce a result similar to (3.19), which implies

$$\frac{\partial^2 (K_{\text{tree}})}{\partial \tau^2} \longrightarrow \frac{1}{2}, \quad \frac{\partial^2 (\delta K_{(g_s)}^{KK})}{\partial \tau^2} \longrightarrow \frac{1}{2} A \sim \frac{1}{2} \frac{g^2}{16\pi^2}, \quad (3.23)$$

and thus

$$\frac{\partial^2 (\delta K_{(g_s)}^{KK})}{\partial \tau^2} \sim \frac{g^2}{16\pi^2} \frac{\partial^2 (K_{\text{tree}})}{\partial \tau^2}. \quad (3.24)$$

Using equation (3.18) we then guess for the scaling behavior of the string loop corrections to the Kähler potential

$$\frac{\partial^2 (\delta K_{(g_s)}^{KK})}{\partial \tau^2} \sim \frac{f(\text{Re}(S))}{16\pi^2} \frac{1}{\tau} \frac{\partial^2 (K_{\text{tree}})}{\partial \tau^2}, \quad (3.25)$$

where we have introduced an unknown function of the dilaton $f(\text{Re}(S))$ representing an integration constant.² However we may be able to use similar reasoning to determine $f(\text{Re}(S))$. The same correction $\delta K_{(g_s)}^{KK}$, upon double differentiation with respect to the dilaton, has to give rise to the 1-loop quantum correction to the corresponding dilaton kinetic term. We also recall that S couples to all field theories on $D3$ -branes as the relative gauge kinetic function is the dilaton itself. Using the same argument as above we end up with the further guess for $\delta K_{(g_s)}^{KK}$:

$$\frac{\partial^2 (\delta K_{(g_s)}^{KK})}{\partial \text{Re}(S)^2} \sim \frac{h(\tau)}{16\pi^2} \frac{1}{\text{Re}(S)} \frac{\partial^2 (K_{\text{tree}})}{\partial \text{Re}(S)^2} \simeq \frac{h(\tau)}{16\pi^2} \frac{1}{\text{Re}(S)^3}, \quad (3.26)$$

where $h(\tau)$ is again an unknown function which parameterises the dependence on the Kähler modulus. Integrating (3.26) twice, we obtain

$$\delta K_{(g_s)}^{KK} \sim \frac{h(\tau)}{16\pi^2} \frac{1}{\text{Re}(S)}, \quad (3.27)$$

²In general there should be also an unknown function of the complex structure and open string moduli but we dropped it since, as we stated at the beginning of this section, its full determination would require an exact string calculation.

where $h(\tau)$ can be worked out from (3.25)

$$\frac{\partial^2 (h(\tau))}{\partial \tau^2} \sim \frac{1}{\tau} \frac{\partial^2 (K_{\text{tree}})}{\partial \tau^2}. \quad (3.28)$$

We now apply the above methods to several Calabi-Yau cases, comparing to either the exact results or the conjecture of equation (3.12)

3.2.1 Case 1: $N=1$ $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

We first consider the case of toroidal compactifications, for which the loop corrections have been explicitly computed [5]. In that case the volume can be expressed as (ignoring the 48 twisted Kähler moduli obtained by blowing up orbifold singularities)

$$\mathcal{V} = \sqrt{\tau_1 \tau_2 \tau_3}, \quad (3.29)$$

and so (3.28) takes the form

$$\frac{\partial^2 (\delta K_{(g_s)}^{KKK})}{\partial \tau_i^2} \sim \frac{f(\text{Re}(S))}{16\pi^2} \frac{1}{\tau_i^3} \quad \forall i = 1, 2, 3. \quad (3.30)$$

Upon integration we get

$$\delta K_{(g_s)}^{KKK} \sim \frac{1}{16\pi^2} \frac{f(\text{Re}(S))}{\tau_i} \quad \forall i = 1, 2, 3. \quad (3.31)$$

Now combining this result with the analysis for the dilatonic dependence of the string loop corrections, we obtain

$$\delta K_{(g_s)}^{KKK} \sim \frac{1}{16\pi^2} \sum_{i=1}^3 \frac{1}{\text{Re}(S)\tau_i}, \quad (3.32)$$

which reproduces the scaling behaviour of the result (3.9) found from string scattering amplitudes.

3.2.2 Case 2: $\mathbb{C}P_{[1,1,1,6,9]}^4$

We next consider loop corrections to the Kähler potential for the Calabi-Yau orientifold $\mathbb{C}P_{[1,1,1,6,9]}^4$. We will compare the form of (3.12) (see also (A.23)) to that arising from our method (3.28) to work out the behaviour of $\delta K_{(g_s)}^{KKK}$, finding again a matching.³ In the large volume limit we can write the volume as follows

$$\mathcal{V} = \frac{1}{9\sqrt{2}} \left(\tau_5^{3/2} - \tau_4^{3/2} \right) \simeq \tau_5^{3/2}, \quad (3.33)$$

and (3.12) becomes

$$\delta K_{(g_s)}^{KKK} \sim \frac{\mathcal{C}_4^{KKK} \sqrt{\tau_4}}{\text{Re}(S) \mathcal{V}} + \frac{\mathcal{C}_5^{KKK} \sqrt{\tau_5}}{\text{Re}(S) \mathcal{V}} \simeq \frac{\mathcal{C}_4^{KKK} \sqrt{\tau_4}}{\text{Re}(S) \mathcal{V}} + \frac{\mathcal{C}_5^{KKK}}{\text{Re}(S) \tau_5}. \quad (3.34)$$

³We note that the topology of $\mathbb{C}P_{[1,1,1,6,9]}^4$ does not allow to have $\delta K_{(g_s)}^W \neq 0$ [16].

From the tree-level Kähler matrix we read

$$\frac{\partial^2 (K_{\text{tree}})}{\partial \tau_4^2} \simeq \frac{1}{\sqrt{\tau_4} \mathcal{V}}, \quad \frac{\partial^2 (K_{\text{tree}})}{\partial \tau_5^2} \simeq \frac{1}{\tau_5^2}. \quad (3.35)$$

Requiring loop corrections to be suppressed by a factor of g_c^2 for the field-theory on the brane gives

$$\begin{cases} \frac{\partial^2 (\delta K_{(g_s)}^{KK})}{\partial \tau_4^2} \sim \frac{1}{16\pi^2} \frac{1}{\text{Re}(S)} \frac{1}{\tau_4^{3/2} \mathcal{V}} \\ \frac{\partial^2 (\delta K_{(g_s)}^{KK})}{\partial \tau_5^2} \sim \frac{1}{16\pi^2} \frac{1}{\text{Re}(S)} \frac{1}{\tau_5^3} \end{cases} \quad (3.36)$$

which, upon double integration, matches exactly the scaling behaviour of the result (3.34).

3.2.3 Case 3: $\mathbb{C}P_{[1,1,2,2,6]}^4$

As another example we study the expected form of loop corrections for the case of the Calabi-Yau manifold $\mathbb{C}P_{[1,1,2,2,6]}^4$, defined by the degree 12 hypersurface embedding. This Calabi-Yau is a K3 fibration and has $(h^{1,1}, h^{2,1}) = (2, 128)$ with $\chi = -252$. Including only the complex structure deformations that survive the mirror map, the defining equation is

$$z_1^{12} + z_2^{12} + z_3^6 + z_4^6 + z_5^2 - 12\psi z_1 z_2 z_3 z_4 z_5 - 2\phi z_1^6 z_2^6 = 0. \quad (3.37)$$

In terms of 2-cycle volumes the overall volume takes the form

$$\mathcal{V} = t_1 t_2^2 + \frac{2}{3} t_2^3, \quad (3.38)$$

giving relations between the 2- and 4-cycle volumes,

$$\begin{aligned} \tau_1 &= t_2^2, & \tau_2 &= 2t_2(t_1 + t_2), \\ t_2 &= \sqrt{\tau_1}, & t_1 &= \frac{\tau_2 - 2\tau_1}{2\sqrt{\tau_1}}, \end{aligned} \quad (3.39)$$

allowing us to write

$$\mathcal{V} = \frac{1}{2} \sqrt{\tau_1} \left(\tau_2 - \frac{2}{3} \tau_1 \right). \quad (3.40)$$

Let us now investigate what the arguments above suggest for the form of the string loop corrections for the $\mathbb{C}P_{[1,1,2,2,6]}^4$ model should look like. Applying (3.12) and (3.13) for the one-loop correction to K , we find

$$\delta K_{(g_s)}^{KK} \sim \frac{\mathcal{C}_1^{KK}}{\text{Re}(S) \mathcal{V}} \frac{\tau_2 - 2\tau_1}{2\sqrt{\tau_1}} + \frac{\mathcal{C}_2^{KK} \sqrt{\tau_1}}{\text{Re}(S) \mathcal{V}}, \quad (3.41)$$

along with

$$\delta K_{(g_s)}^W \sim \frac{\mathcal{C}_1^W}{\mathcal{V}} \frac{2\sqrt{\tau_1}}{\tau_2 - 2\tau_1} + \frac{\mathcal{C}_2^W}{\mathcal{V} \sqrt{\tau_1}}. \quad (3.42)$$

The arguments summarized in the relation (3.28) reproduce exactly the behaviour of these corrections. The tree-level Kähler metric reads

$$\frac{\partial^2 (K_{\text{tree}})}{\partial \tau_1^2} = \frac{1}{\tau_1^2} + \frac{2}{9} \frac{\tau_1}{\mathcal{V}^2}, \quad \frac{\partial^2 (K_{\text{tree}})}{\partial \tau_2^2} = \frac{1}{2} \frac{\tau_1}{\mathcal{V}^2}. \quad (3.43)$$

Given that we are interested simply in the scaling behaviour of these corrections, we notice that either in the case $\tau_1 \lesssim \tau_2$ such that

$$\mathcal{V} = \frac{1}{2}\sqrt{\tau_1} \left(\tau_2 - \frac{2}{3}\tau_1 \right) \simeq \tau_1^{3/2} \simeq \tau_2^{3/2}, \quad (3.44)$$

or in the large volume limit $\tau_1 \ll \tau_2$ where

$$\mathcal{V} \simeq \sqrt{\tau_1}\tau_2, \quad (3.45)$$

the matrix elements (3.43) take the form

$$\frac{\partial^2 (K_{\text{tree}})}{\partial \tau_1^2} \sim \frac{1}{\tau_1^2}, \quad \frac{\partial^2 (K_{\text{tree}})}{\partial \tau_2^2} \sim \frac{1}{\tau_2^2}. \quad (3.46)$$

We can now see that our method (3.28) yields

$$\left\{ \begin{array}{l} \frac{\partial^2 (\delta K_{(g_s)}^{KKK})}{\partial \tau_1^2} \sim \frac{1}{16\pi^2} \frac{1}{\text{Re}(S)\tau_1} \frac{\partial^2 (K_{\text{tree}})}{\partial \tau_1^2} \iff \delta K_{(g_s, \tau_1)}^{KKK} \sim \frac{1}{\text{Re}(S)\tau_1} \\ \frac{\partial^2 (\delta K_{(g_s)}^{KKK})}{\partial \tau_2^2} \sim \frac{1}{16\pi^2} \frac{1}{\text{Re}(S)\tau_2} \frac{\partial^2 (K_{\text{tree}})}{\partial \tau_2^2} \iff \delta K_{(g_s, \tau_2)}^{KKK} \sim \frac{1}{\text{Re}(S)\tau_2} \end{array} \right. \quad (3.47)$$

which, both in the case $\tau_1 \lesssim \tau_2$ and $\tau_1 \ll \tau_2$, matches the scaling behaviour of (3.41)

$$\delta K_{(g_s)}^{KKK} \sim \frac{\mathcal{C}_1^{KK}}{\text{Re}(S)\mathcal{V}} \frac{\tau_2 - 2\tau_1}{2\sqrt{\tau_1}} + \frac{\mathcal{C}_2^{KK}\sqrt{\tau_1}}{\text{Re}(S)\mathcal{V}} \sim \frac{\mathcal{C}_1^{KK}}{\text{Re}(S)\tau_1} + \frac{\mathcal{C}_2^{KK}}{\text{Re}(S)\tau_2}. \quad (3.48)$$

4. Extended no scale structure

The examples in the previous section give support to the notion that loop corrections to the Kähler potential can be understood by requiring that the loop-corrected kinetic terms for a modulus τ are suppressed by a factor of g^2 for the gauge group on branes wrapping the τ cycle. We repeat again that these arguments only apply to moduli that control loop factors.

While not proven, we now assume the validity of this parametric form of the corrections and move on to study the effect of such corrections in the scalar potential. We shall show that the leading contribution to the scalar potential is null, due to a cancellation in the expression for the scalar potential. We shall see that this cancellation holds so long as $\delta K_{(g_s)}^{KKK}$ is an homogeneous function of degree $n = -2$ in the 2-cycle volumes. We call this “extended no scale structure”, as the cancellation in the scalar potential that is characteristic of no-scale models extends to one further order, so that compared to a naive expectation the scalar potential is only non-vanishing at sub-sub-leading order. Let us state clearly the “extended no-scale structure” result:

Let X be a Calabi-Yau three-fold and consider type IIB $N = 1$ 4D SUGRA where the Kähler potential and the superpotential in the Einstein frame take the form:

$$\left\{ \begin{array}{l} K = K_{\text{tree}} + \delta K, \\ W = W_0. \end{array} \right. \quad (4.1)$$

If and only if the loop correction δK to K is a homogeneous function in the 2-cycles volumes of degree $n = -2$, then at leading order

$$\delta V_{(g_s)} = 0. \quad (4.2)$$

We shall provide now a rigorous proof of the previous claim. We are interested only in the perturbative part of the scalar potential. We therefore focus on

$$\delta V_{(g_s)} = (K^{ij} \partial_i K \partial_j K - 3) \frac{|W|^2}{\mathcal{V}^2}, \quad (4.3)$$

where $K = -2 \ln(\mathcal{V}) + \delta K_{(g_s)}$. We focus on δK coming from g_s (rather than α') corrections. We require the inverse of the quantum corrected Kähler matrix, which can be found using the Neumann series. Introducing an expansion parameter ε , and writing K_{tree} as K_0 , we define

$$\mathcal{K}_0 = \left\{ \frac{\partial^2 K_0}{\partial \tau_i \partial \tau_j} \right\}_{i,j=1,\dots,h_{1,1}}, \quad \delta \mathcal{K} = \left\{ \frac{\partial^2 (\delta K_{(g_s)})}{\partial \tau_i \partial \tau_j} \right\}_{i,j=1,\dots,h_{1,1}} \quad (4.4)$$

and have

$$K^{ij} = (\mathcal{K}_0 + \varepsilon \delta \mathcal{K})^{ij} = (\mathcal{K}_0 (\mathbf{1} + \varepsilon \mathcal{K}_0^{-1} \delta \mathcal{K}))^{ij} = (\mathbf{1} + \varepsilon \mathcal{K}_0^{-1} \delta \mathcal{K})^{il} K_0^{lj}. \quad (4.5)$$

Now use the Neumann series

$$(\mathbf{1} + \varepsilon \mathcal{K}_0^{-1} \delta \mathcal{K})^{il} = \delta_l^i - \varepsilon K_0^{im} \delta K_{ml} + \varepsilon^2 K_0^{im} \delta K_{mp} K_0^{pq} \delta K_{ql} + \mathcal{O}(\varepsilon^3), \quad (4.6)$$

to find

$$K^{ij} = K_0^{ij} - \varepsilon K_0^{im} \delta K_{ml} K_0^{lj} + \varepsilon^2 K_0^{im} \delta K_{mp} K_0^{pq} \delta K_{ql} K_0^{lj} + \mathcal{O}(\varepsilon^3). \quad (4.7)$$

Substituting (4.7) back in (4.3), we obtain

$$\delta V_{(g_s)} = V_0 + \varepsilon \delta V_1 + \varepsilon^2 \delta V_2 + \mathcal{O}(\varepsilon^3), \quad (4.8)$$

where $V_0 = (K_0^{ij} K_i^0 K_j^0 - 3) \frac{|W|^2}{\mathcal{V}^2} = 0$ due to (2.12) is the usual no-scale structure and

$$\begin{cases} \delta V_1 = \left(2K_0^{ij} K_i^0 \delta K_j - K_0^{im} \delta K_{ml} K_0^{lj} K_i^0 K_j^0 \right) \frac{|W|^2}{\mathcal{V}^2} \\ \delta V_2 = \left(K_0^{ij} \delta K_i \delta K_j - 2K_0^{im} \delta K_{ml} K_0^{lj} K_i^0 \delta K_j \right. \\ \quad \left. + K_0^{im} \delta K_{mp} K_0^{pq} \delta K_{ql} K_0^{lj} K_i^0 K_j^0 \right) \frac{|W|^2}{\mathcal{V}^2}. \end{cases} \quad (4.9)$$

We caution the reader that (4.8) is not a loop expansion of the scalar potential but rather an expansion of the scalar potential arising from the 1-loop quantum corrected Kähler metric. The statement of extended no-scale structure is that δV_1 will vanish, while δV_2 will be non-vanishing. Recalling (2.11), δV_1 simplifies to

$$\delta V_1 = - \left(2\tau_j \frac{\partial (\delta K)}{\partial \tau_j} + \tau_m \tau_l \frac{\partial^2 (\delta K)}{\partial \tau_m \partial \tau_l} \right) \frac{|W|^2}{\mathcal{V}^2}. \quad (4.10)$$

Let us make a change of coordinates and work with the 2-cycle volumes instead of the 4-cycles. Using the second of the relations (2.7), we deduce

$$2\tau_j \frac{\partial}{\partial \tau_j} = t_l \frac{\partial}{\partial t_l}, \quad (4.11)$$

and

$$\tau_m \tau_l \frac{\partial^2}{\partial \tau_m \partial \tau_l} = \frac{1}{4} t_i t_k \frac{\partial^2}{\partial t_i \partial t_k} + \frac{1}{4} A_{li} t_i t_k \frac{\partial (A^{lp})}{\partial t_k} \frac{\partial}{\partial t_p}. \quad (4.12)$$

From the definition (2.6) of A_{li} , we notice that A_{li} is an homogeneous function of degree $n = 1 \forall l, i$. Inverting the matrix, we still get homogeneous matrix elements but now of degree $n = -1$. Finally the Euler theorem for homogeneous functions, tells us that

$$t_k \frac{\partial (A^{lp})}{\partial t_k} = (-1) A^{lp}, \quad (4.13)$$

which gives

$$\tau_m \tau_l \frac{\partial^2}{\partial \tau_m \partial \tau_l} = \frac{1}{4} t_i t_k \frac{\partial^2}{\partial t_i \partial t_k} - \frac{1}{4} t_p \frac{\partial}{\partial t_p}, \quad (4.14)$$

and, in turn

$$\delta V_1 = -\frac{1}{4} \left(3t_l \frac{\partial (\delta K)}{\partial t_l} + t_i t_k \frac{\partial^2 (\delta K)}{\partial t_i \partial t_k} \right) \frac{|W|^2}{\mathcal{V}^2}. \quad (4.15)$$

The form of equation (3.12) suggests that for arbitrary Calabi-Yaus the string loop corrections to K will be homogeneous functions of the 2-cycle volumes, and in particular that the leading correction will be of degree -2 in 2-cycle volumes. Therefore if the degree of δK is n , the Euler theorem tells us that

$$\delta V_1 = -\frac{|W|^2}{\mathcal{V}^2} \frac{1}{4} (3n + n(n-1)) \delta K = -\frac{|W|^2}{\mathcal{V}^2} \frac{1}{4} n(n+2) \delta K. \quad (4.16)$$

It follows then, as we claimed above, that only $n = -2$ implies $\delta V_1 = 0$. In particular, from the conjectures (3.12) and (3.13), we see that

$$\begin{cases} n = -2 & \text{for } \delta K_{(g_s)}^{KK}, \\ n = -4 & \text{for } \delta K_{(g_s)}^W, \end{cases} \quad (4.17)$$

and so

$$\begin{cases} \delta V_{(g_s),1}^{KK} = 0, \\ \delta V_{(g_s),1}^W = -2 \delta K_{(g_s)}^W \frac{|W|^2}{\mathcal{V}^2}. \end{cases} \quad (4.18)$$

4.1 General formula for the effective scalar potential

Let us now work out the general formula for the effective scalar potential evaluating also the first non-vanishing contribution of $\delta K_{(g_s)}^{KK}$, that is the ε^2 terms (4.9) in V

$$\begin{aligned} \delta V_2 = & \left(K_0^{ij} \delta K_i \delta K_j - 2K_0^{im} \delta K_{ml} K_0^{lj} K_i^0 \delta K_j \right. \\ & \left. + K_0^{im} \delta K_{mp} K_0^{pq} \delta K_{ql} K_0^{lj} K_i^0 K_j^0 \right) \frac{|W|^2}{\mathcal{V}^2}. \end{aligned} \quad (4.19)$$

Using (2.11), δV_2 simplifies to

$$\delta V_2 = \left(K_0^{ij} \delta K_i \delta K_j + 2\tau_m \delta K_{ml} K_0^{lj} \delta K_j + \tau_m \tau_q \delta K_{ml} K_0^{lp} \delta K_{pq} \right) \frac{|W|^2}{\mathcal{V}^2}. \quad (4.20)$$

We now stick to the case where $\delta K_{(g_s)}^{KK}$ is given by the conjecture (3.12). Considering just the contribution from one modulus (as the contributions from different terms are independent), and dropping the dilatonic dependence, we have

$$\delta K \rightarrow \delta K_{(g_s), \tau_a}^{KK} \sim \frac{\mathcal{C}_a^{KK} t_a}{\mathcal{V}}. \quad (4.21)$$

From (4.21) we notice that

$$\begin{aligned} \delta K_m &= A^{mj} \frac{\partial(\delta K)}{\partial t^j} = \mathcal{C}_a^{KK} A^{mj} \left(-\frac{t_a}{\mathcal{V}^2} \frac{\partial(\mathcal{V})}{\partial t^j} + \frac{\delta_{aj}}{\mathcal{V}} \right) \\ &= \mathcal{C}_a^{KK} \left(-\frac{1}{2} \frac{t_a t_m}{\mathcal{V}^2} + \frac{A^{am}}{\mathcal{V}} \right) = -\mathcal{C}_a^{KK} K_{am}^0, \end{aligned} \quad (4.22)$$

thus

$$K_0^{ij} \delta K_j = -\mathcal{C}_a^{KK} K_0^{ij} K_{aj}^0 = -\mathcal{C}_a^{KK} \delta_{ai}. \quad (4.23)$$

With this consideration (4.20) becomes

$$\delta V_2 = \left(-\mathcal{C}_a^{KK} \delta K_a - 2\mathcal{C}_a^{KK} \tau_m \delta K_{ma} + \tau_m \tau_q \delta K_{ml} K_0^{lp} \delta K_{pq} \right) \frac{|W|^2}{\mathcal{V}^2}. \quad (4.24)$$

We need now to evaluate

$$\tau_m \delta K_{ml} = \frac{1}{2} t_p \frac{\partial}{\partial t^p} \left(A^{li} \frac{\partial(\delta K)}{\partial t^i} \right) = \frac{1}{2} t_p \frac{\partial}{\partial t^p} (\delta K_l) = -2\delta K_l, \quad (4.25)$$

that yields

$$\delta V_2 = \left(-\mathcal{C}_a^{KK} \delta K_a + 4\mathcal{C}_a^{KK} \delta K_a + 4\delta K_l K_0^{lp} \delta K_p \right) \frac{|W|^2}{\mathcal{V}^2} \quad (4.26)$$

$$= \left(-\mathcal{C}_a^{KK} \delta K_a + 4\mathcal{C}_a^{KK} \delta K_a - 4\mathcal{C}_a^{KK} \delta K_a \right) \frac{|W|^2}{\mathcal{V}^2} \quad (4.27)$$

$$= -\mathcal{C}_a^{KK} \delta K_a \frac{|W|^2}{\mathcal{V}^2}. \quad (4.28)$$

With the help of the relation (4.22) and replacing the dilatonic dependence, we can write the previous expression in terms of the tree-level Kähler metric

$$\delta V_2 = \frac{(\mathcal{C}_a^{KK})^2}{\text{Re}(S)^2} K_{aa}^0 \frac{|W|^2}{\mathcal{V}^2}. \quad (4.29)$$

Putting together (4.18) and (4.29), we can now write the quantum correction to the scalar potential at leading order at 1 loop for general Calabi-Yaus, in terms of the cycles i wrapped by the branes and the quantum corrections to the Kähler potential,

$$\delta V_{(g_s)}^{1loop} = \sum_i \left(\frac{(\mathcal{C}_i^{KK})^2}{\text{Re}(S)^2} K_{ii}^0 - 2\delta K_{(g_s), \tau_i}^W \right) \frac{W_0^2}{\mathcal{V}^2}. \quad (4.30)$$

We emphasise that this formula assumes the validity of the BHP conjecture, and only focuses on corrections of this nature.

Finally we point out that, due to the extended no-scale structure, in the presence of non-perturbative contributions to the superpotential, it is also important to check that the leading quantum corrections to the general scalar potential (2.24) are indeed given by (4.30) and the contribution to the non-perturbative part of the scalar potential generated by string loop corrections

$$\delta V_{np} = \left(2K_0^{ij} W_i \delta K_{(g_s),j} W + \delta K_{(g_s)}^{ij} W_i W_j \right) \frac{|W|^2}{\mathcal{V}^2}, \quad (4.31)$$

is irrelevant. A quick calculation shows that this is indeed the case.⁴

4.2 Field theory interpretation

We now interpret the above arguments and in particular the existence of the extended no-scale structure in light of the Coleman-Weinberg potential [17].⁵ We will see that this gives a quantitative explanation for the cancellation that is present. The Coleman-Weinberg potential is given in supergravity by (e.g. see [19])

$$\delta V_{1loop} = \frac{1}{64\pi^2} \left[\Lambda^4 STr(M^0) \ln\left(\frac{\Lambda^2}{\mu^2}\right) + 2\Lambda^2 STr(M^2) + STr\left(M^4 \ln\left(\frac{M^2}{\Lambda^2}\right)\right) \right], \quad (4.32)$$

where μ is a scale parameter, Λ the cut-off scale and

$$STr(M^n) \equiv \sum_i (-1)^{2j_i} (2j_i + 1) m_i^n, \quad (4.33)$$

is the supertrace, written in terms of the the spin of the different particles j_i and the field-dependent mass eigenvalues m_i .

The form of (4.32) gives a field theory interpretation to the scalar potential found in section 4.1. Let us try and match the 1-loop expression with the potential (4.32) interpreting the various terms in the Coleman-Weinberg potential as different terms in the ϵ expansion in (4.8). We first notice that in any spontaneously broken supergravity theory, $STr(M^0) = 0$, as the number of bosonic and fermionic degrees of freedom must be equal. The leading term in (4.32) is therefore null.

We recall that due to the extended no-scale structure the coefficient of the $\mathcal{O}(\epsilon)$ term in (4.8) is also vanishing. Our comparison should therefore involve the leading non-zero terms in both cases. In the following paragraphs, we will re-analyse the three examples studied in section 3 and show how we always get a matching. This gives a nice physical understanding of this cancellation at leading order in $\delta V_{(g_s),1-loop}^{KK}$ which is due just to supersymmetry: the cancellation must take place if the resulting 1-loop potential is to

⁴We shall not discuss the effects of higher loop contributions to the scalar potential. We expect that these will be suppressed compared to the one-loop contribution by additional loop factors of $(16\pi^2)$, and so will not compete with the terms considered in (4.30).

⁵For a previous attempt at matching string effective actions onto the Coleman-Weinberg potential, see [18].

match onto the Coleman-Weinberg form. Supersymmetry causes the vanishing of the first term in (4.32) and we notice, for each example, that the second term in (4.32) scales as the $\mathcal{O}(\epsilon^2)$ term in (4.8), therefore, in order to match the two results, the $\mathcal{O}(\epsilon)$ term in (4.8) also has to be zero. This is, in fact, what the extended no-scale structure guarantees.

We note here that both, with the use of the supergravity expression for the Coleman-Weinberg formula, and for the earlier discussions of section 3, supersymmetry has played a crucial role. In the Coleman-Weinberg formula, the presence of low-energy supersymmetry is used to evaluate the supertraces and to relate these to the gravitino mass. In the discussion of kinetic terms, the fact that the corrections are written as corrections to the Kähler metric automatically implies that the structure of low-energy supersymmetry is respected.

4.2.1 Case 1: $N=1$ $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

The case of the $N=1$ toroidal orientifold background was studied in section 3.1.1 and 3.2.1. We here treat all three moduli on equal footing, reducing the volume form (3.29) to the one-modulus case

$$\mathcal{V} = \tau^{3/2} = \left(\frac{T + \bar{T}}{2} \right)^{3/2}. \tag{4.34}$$

We therefore take

$$\langle \tau_1 \rangle \simeq \langle \tau_2 \rangle \simeq \langle \tau_3 \rangle. \tag{4.35}$$

We write out very explicitly the correction to the scalar potential due to the correction to the Kähler potential as computed in [5]. We focus only on the Kähler moduli dependence. The tree level Kähler potential is

$$K = -3 \ln(T + \bar{T})$$

and the loop-corrected Kähler potential has the form

$$K = -3 \ln(T + \bar{T}) + \frac{\epsilon}{(T + \bar{T})}.$$

The scalar potential is

$$V = M_P^4 e^K \left(K^{i\bar{j}} \partial_i K \partial_{\bar{j}} K - 3 \right) |W|^2.$$

Evaluated, this gives

$$\begin{aligned} V &= \frac{M_P^4}{(T + \bar{T})^3} \left(0 + \frac{0 \times \mathcal{O}(\epsilon)}{T + \bar{T}} + \frac{\mathcal{O}(\epsilon^2)}{(T + \bar{T})^2} \right) \\ &= \frac{M_P^4 \epsilon^2}{(T + \bar{T})^5} \sim \frac{M_P^4 \epsilon^2}{\mathcal{V}^{10/3}}. \end{aligned} \tag{4.36}$$

The cancellation of the $\mathcal{O}(T + \bar{T})^{-3}$ term in (4.36) is due to the original no-scale structure. The cancellation of the $\mathcal{O}(T + \bar{T})^{-4}$ term in (4.36) is due to the extended no-scale structure that is satisfied by the loop corrected Kähler potential, giving a leading contribution at

$\mathcal{O}(T + \bar{T})^{-5}$. This gives the behaviour of the leading contribution to the scalar potential, which we want to compare with the Coleman-Weinberg expression.

To compare with (4.32) we recall that in supergravity the supertrace is proportional to the gravitino mass:

$$STr(M^2) \simeq m_{3/2}^2. \quad (4.37)$$

The dependence of the gravitino mass on the volume is always given by

$$m_{3/2}^2 = e^K W_0^2 \simeq \frac{1}{\mathcal{V}^2} \implies STr(M^2) \simeq \frac{1}{\mathcal{V}^2}. \quad (4.38)$$

We must also understand the scaling behaviour of the cut-off Λ . Λ should be identified with the energy scale above which the four-dimensional effective field theory breaks down. This is the compactification scale at which many new KK states appear, and so is given by

$$\Lambda = m_{KK} \simeq \frac{M_s}{R} = \frac{M_s}{\tau^{1/4}} = \frac{1}{\tau^{1/4}} \frac{1}{\sqrt{\mathcal{V}}} M_P = \frac{M_P}{\mathcal{V}^{2/3}}. \quad (4.39)$$

In units of the Planck mass, (4.32) therefore scales as

$$\begin{aligned} \delta V_{1loop} &\simeq 0 \cdot \Lambda^4 + \Lambda^2 STr(M^2) + STr\left(M^4 \ln\left(\frac{M^2}{\Lambda^2}\right)\right) \simeq \\ &\simeq 0 \cdot \frac{1}{\mathcal{V}^{8/3}} + \frac{1}{\mathcal{V}^{10/3}} + \frac{1}{\mathcal{V}^4}, \end{aligned} \quad (4.40)$$

in agreement with (4.36).

4.2.2 Case 2: $\mathbb{C}P^4_{[1,1,1,6,9]}$

This case, studied in section 3.2.2, is more involved, as it includes two Kähler moduli, the large modulus $\tau_b \simeq \mathcal{V}^{2/3}$ and the small modulus τ_s . The effective potential gets contributions from loop corrections for both moduli and in these two cases, (4.3) takes the form (the dilaton is considered fixed and its dependence is reabsorbed in \mathcal{C}_b^{KK} and \mathcal{C}_s^{KK})

1. Big modulus

$$\begin{aligned} \delta V_{(g_s),1-loop}^{KK} &= \left(0 \cdot \frac{\mathcal{C}_b^{KK}}{\tau_b} + \frac{\alpha_{2,b} (\mathcal{C}_b^{KK})^2}{\tau_b^2} + \frac{\alpha_{3,b} (\mathcal{C}_b^{KK})^3}{\tau_b^3} + \mathcal{O}\left(\frac{\partial^4 K_0}{\partial \tau_b^4}\right) \right) \frac{W_0^2}{\mathcal{V}^2} \\ &\simeq \left(0 \cdot \frac{\mathcal{C}_b^{KK}}{\mathcal{V}^{8/3}} + \frac{\alpha_{2,b} (\mathcal{C}_b^{KK})^2}{\mathcal{V}^{10/3}} + \frac{\alpha_{3,b} (\mathcal{C}_b^{KK})^3}{\mathcal{V}^4} \right) W_0^2. \end{aligned} \quad (4.41)$$

2. Small modulus

$$\delta V_{(g_s),1-loop}^{KK} = \left(0 \cdot \mathcal{C}_s^{KK} \frac{\sqrt{\tau_s}}{\mathcal{V}^3} + \frac{\alpha_{2,s} (\mathcal{C}_s^{KK})^2}{\mathcal{V}^3 \sqrt{\tau_s}} + \frac{\alpha_{3,s} (\mathcal{C}_s^{KK})^3}{\mathcal{V}^3 \tau_s^{3/2}} + \mathcal{O}\left(\mathcal{V}^{-2} \frac{\partial^4 K_0}{\partial \tau_s^4}\right) \right) W_0^2. \quad (4.42)$$

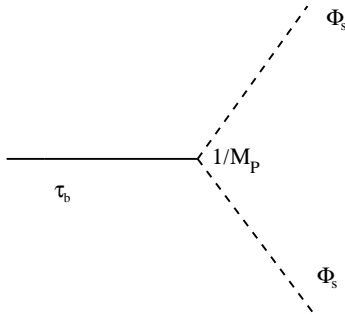


Figure 2: Coupling of the big modulus KK modes to a generic field Φ_s living on the brane wrapping the small 4-cycle.

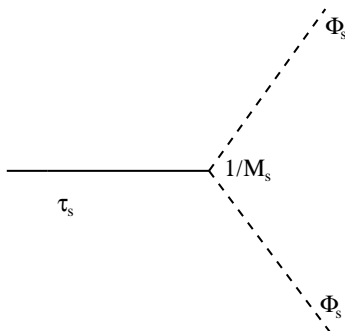


Figure 3: Coupling of the small modulus KK modes to a generic field Φ_s living on the brane wrapping the small 4-cycle.

In the Coleman-Weinberg potential, the supertrace has the same scaling $\sim \mathcal{V}^{-2}$ as in (4.38), but there now exist different values of the cut-off Λ for the field theories living on branes wrapping the big and small 4-cycles

$$\begin{cases} \Lambda_b = m_{KK,b} \simeq \frac{1}{\tau_b^{1/4}} \frac{1}{\sqrt{\mathcal{V}}} M_P = \frac{M_P}{\mathcal{V}^{2/3}}, \\ \Lambda_s = m_{KK,s} \simeq \frac{1}{\tau_s^{1/4}} \frac{1}{\sqrt{\mathcal{V}}} M_P. \end{cases} \quad (4.43)$$

The existence of two cut-off scales requires some explanation. At first glance, as $\Lambda_b < \Lambda_s$ and the KK modes of the big Kähler modulus couple to the field theory on the brane wrapping the small 4-cycle, one might think that there is just one value of the cut-off Λ , which is given by $\Lambda_b = m_{KK,b}$. This corresponds to the mass scale of the lowest Kaluza-Klein mode present in the theory. For a field theory living on a brane wrapping the large cycle, this represents the mass scale of Kaluza-Klein replicas of the gauge bosons and matter fields of the theory. However, we do not think this is the correct interpretation for a field theory living on the small cycle. The bulk Kaluza-Klein modes are indeed lighter than those associated with the small cycle itself.

However it is also the case that the bulk modes couple extremely weakly to this field theory compared to the local modes. The bulk modes only couple gravitationally to this

field theory, whereas the local modes couple at the string scale [21]. In the case that the volume is extremely large, this difference is significant. For a field theory on the small cycle, the cutoff should be the scale at which KK replicas of the quarks and gluons appear, rather than the scale at which new very weakly coupled bulk modes are present. As the local modes are far more strongly coupled, it is these modes that determine the scale of the UV cutoff. This is illustrated in figure 2 and 3.⁶

We now move on to make the matching of (4.41) and (4.42) with the Coleman-Weinberg potential (4.32). For the big modulus, we find

$$\begin{aligned} \delta V_{1loop} &\simeq 0 \cdot \Lambda_b^4 + \Lambda_b^2 STr(M^2) + STr\left(M^4 \ln\left(\frac{M^2}{\Lambda_b^2}\right)\right) \\ &\simeq 0 \cdot \frac{1}{\mathcal{V}^{8/3}} + \frac{1}{\mathcal{V}^{10/3}} + \frac{1}{\mathcal{V}^4}, \end{aligned} \tag{4.44}$$

which yields again a scaling matching that of (4.41). For the small modulus we obtain, proceeding as in the previous case

$$\begin{aligned} \delta V_{1loop} &\simeq 0 \cdot \Lambda_s^4 + \Lambda_s^2 STr(M^2) + STr\left(M^4 \ln\left(\frac{M^2}{\Lambda_s^2}\right)\right) \\ &\simeq 0 \cdot \frac{1}{\tau_s} \frac{1}{\mathcal{V}^2} + \frac{1}{\sqrt{\tau_s}} \frac{1}{\mathcal{V}^3} + \frac{1}{\mathcal{V}^4}, \end{aligned} \tag{4.45}$$

where we have a matching only of the second term of (4.45) with the second term of (4.42). This is indeed the term which we expect to match, given that is the first non-vanishing leading contribution to the effective scalar potential at 1-loop. There is no reason the first terms need to match as they have vanishing coefficients.

As an aside, we finally note that the third term in (4.42) can also match with the Coleman-Weinberg effective potential, although we should not try to match this with the third term in (4.32) but with a subleading term in the expansion of the second term in (4.32). This is due to the fact that we do not have full control on the expression for the Kaluza-Klein scale (4.43). In the presence of fluxes, this is more reasonably given by (for example see the discussion in appendix D of [6])

$$\begin{aligned} \Lambda_s &= m_{KK,s} \simeq \frac{1}{\tau_s^{1/4}} \frac{M_P}{\sqrt{\mathcal{V}}} \left(1 + \frac{1}{\tau_s} + \dots\right) = \frac{1}{\tau_s^{1/4}} \frac{M_P}{\sqrt{\mathcal{V}}} + \frac{1}{\tau_s^{5/4}} \frac{M_P}{\sqrt{\mathcal{V}}} + \dots \\ \implies \Lambda_s^2 &\simeq \frac{1}{\tau_s^{1/2}} \frac{M_P^2}{\mathcal{V}} + \frac{2}{\tau_s^{3/2}} \frac{M_P^2}{\mathcal{V}}. \end{aligned} \tag{4.46}$$

This, in turn, produces

$$\Lambda_s^2 STr(M^2) \simeq \frac{1}{\tau_s^{1/2}} \frac{1}{\mathcal{V}^3} + \frac{2}{\tau_s^{3/2}} \frac{1}{\mathcal{V}^3}. \tag{4.47}$$

In this case the second term in (4.47) reproduces the scaling behaviour of the third term in (4.42).

⁶Notice that the cut-off dependence of the $STr(M^2)$ term could potentially be dangerous for the stability of the magnitude of soft terms computed for this model in references [22]. With our analysis here it is easy to see that the contribution of this term to the scalar potential and then to the structure of soft breaking terms is suppressed by inverse powers of the volume and is therefore harmless.

4.2.3 Case 3: $\mathbb{C}P^4_{[1,1,2,2,6]}$

In section 3.2.3 we have seen that there are two regimes where the case of the $K3$ Fibration with two Kähler moduli can be studied. When the VEVs of the two moduli are of the same order of magnitude, they can be treated on equal footing and the volume form which, as we have just seen in section 4.2.1, gives also the scaling behaviour of the toroidal orientifold case. We do not need therefore to repeat the same analysis and we automatically know that the scaling of our general result for the effective scalar potential at 1-loop matches exactly the Coleman-Weinberg formula also in this case.

The second situation when $\tau_2 \gg \tau_1$ is more interesting. The relations (3.39) tell us that the large volume limit $\tau_2 \gg \tau_1$ is equivalent to $t_1 \gg t_2$ and thus they reduce to

$$\tau_1 = t_2^2, \quad \tau_2 \simeq 2t_2t_1, \quad \mathcal{V} \simeq \frac{1}{2}\sqrt{\tau_1}\tau_2 \simeq t_1t_2^2. \quad (4.48)$$

The KK scale of the compactification is then set by the large 2-cycle t_1 ,

$$m_{KK} \sim \frac{M_s}{\sqrt{t_1}} \sim \frac{M_P}{t_1t_2}, \quad (4.49)$$

while in the large volume limit the gravitino mass is

$$m_{3/2} \sim \frac{M_P}{\mathcal{V}} \sim \frac{M_P}{t_1t_2^2}. \quad (4.50)$$

The bulk KK scale is therefore comparable to that of the gravitino mass, and it is not clear that this limit can be described in the language of four-dimensional supergravity. Let us nonetheless explore the consequences of using the same analysis as in the previous sections. The evaluation of (4.8) gives (reabsorbing the VEV of the dilaton in \mathcal{C}_1^{KK} and \mathcal{C}_2^{KK})

1. Small modulus τ_1

$$\delta V_{(g_s),1loop}^{KK} \simeq \left(0 \cdot \frac{\mathcal{C}_1^{KK}}{\tau_1 \mathcal{V}^2} + \frac{\alpha_{2,1} (\mathcal{C}_1^{KK})^2}{\tau_1^2 \mathcal{V}^2} + \frac{\alpha_{3,1} (\mathcal{C}_1^{KK})^3}{\tau_1^3 \mathcal{V}^2} \right) W_0^2. \quad (4.51)$$

2. Big modulus τ_2

$$\delta V_{(g_s),1loop}^{KK} \simeq \left(0 \cdot \mathcal{C}_2^{KK} \frac{\sqrt{\tau_1}}{\mathcal{V}^3} + \alpha_{2,2} (\mathcal{C}_2^{KK})^2 \frac{\tau_1}{\mathcal{V}^4} + \alpha_{3,2} (\mathcal{C}_2^{KK})^3 \frac{\tau_1^{3/2}}{\mathcal{V}^5} \right) W_0^2. \quad (4.52)$$

Let us now derive the two different values of the cut-off Λ for the field theories living on branes wrapping the big and small 4-cycles. We realise that the Kaluza-Klein radii for the two field theories on τ_1 and τ_2 are given by

$$\begin{cases} R_1 \simeq \sqrt{t_2}, \\ R_2 \simeq \sqrt{t_1}, \end{cases} \quad (4.53)$$

and consequently

$$\begin{cases} \Lambda_1 = m_{KK,1} \simeq \frac{M_s}{\sqrt{t_2}} \simeq \frac{1}{\tau_1^{1/4} \sqrt{\mathcal{V}}} M_P, \\ \Lambda_2 = m_{KK,2} \simeq \frac{M_s}{\sqrt{t_1}} \simeq \frac{\sqrt{\tau_1}}{\mathcal{V}} M_P. \end{cases} \quad (4.54)$$

We note that $m_{KK,2}$ coincides with the scale of the lightest KK modes m_{KK} . If we try to match the result (4.51) for the small cycle with the corresponding Coleman-Weinberg potential for the field theory on τ_1

$$\begin{aligned} \delta V_{1loop} &\simeq 0 \cdot \Lambda_1^4 + \Lambda_1^2 \text{STr}(M^2) + \text{STr}\left(M^4 \ln\left(\frac{M^2}{\Lambda_1^2}\right)\right) \\ &\simeq 0 \cdot \frac{1}{\tau_1^2 \mathcal{V}^2} + \frac{1}{\sqrt{\tau_1} \mathcal{V}^3} + \frac{1}{\mathcal{V}^4}, \end{aligned} \tag{4.55}$$

we do not find any agreement. This is not surprising since effective field theory arguments only make sense when

$$\delta V_{(g_s),1loop}^{KK} \ll m_{KK}^4, \tag{4.56}$$

but this condition is not satisfied in our case. In fact, using the mass of the lowest KK mode present in the theory, we have

$$m_{KK}^4 = m_{KK,2}^4 \simeq \frac{\tau_1^2}{\mathcal{V}^4} \ll \frac{1}{\tau_1^2 \mathcal{V}^2} \simeq \delta V_{(g_s),1loop}^{KK}. \tag{4.57}$$

Energy densities couple universally through gravity, and so this implies an excitation of Kaluza-Klein modes, taking us beyond the regime of validity of effective field theory. Thus in this limit the use of the four-dimensional supergravity action with loop corrections to compute the effective potential does not seem trustworthy, as it gives an energy density much larger than m_{KK}^4 .

For the field theory on the large cycle τ_2 the Coleman-Weinberg potential gives

$$\begin{aligned} \delta V_{1loop} &\simeq 0 \cdot \Lambda_2^4 + \Lambda_2^2 \text{STr}(M^2) + \text{STr}\left(M^4 \ln\left(\frac{M^2}{\Lambda_2^2}\right)\right) \\ &\simeq 0 \cdot \frac{\tau_1^2}{\mathcal{V}^4} + \frac{\tau_1}{\mathcal{V}^4} + \frac{1}{\mathcal{V}^4}. \end{aligned} \tag{4.58}$$

In this case the energy density given by the loop corrections (4.52) is (marginally) less than $m_{KK}^4 \simeq \tau_1^2 \mathcal{V}^{-4}$, being smaller by a factor of τ_1 . Equation (4.58) then matches the result (4.52) at leading order.

Again, we also note as an aside that if we expand the KK scale as in in section 4.2.2, then we obtain

$$\begin{aligned} \Lambda_2 &= m_{KK,2} \simeq \frac{\sqrt{\tau_1}}{\mathcal{V}} \left(1 + \frac{1}{\tau_2} + \dots\right) M_P \simeq \left(\frac{\sqrt{\tau_1}}{\mathcal{V}} + \frac{\tau_1}{\mathcal{V}^2}\right) M_P \\ \implies \Lambda_2^2 &\simeq \left(\frac{\tau_1}{\mathcal{V}^2} + \frac{\tau_1^{3/2}}{\mathcal{V}^3}\right) M_P^2. \end{aligned} \tag{4.59}$$

This, in turn, produces

$$\Lambda_2^2 \text{STr}(M^2) \simeq \frac{\tau_1}{\mathcal{V}^4} + \frac{\tau_1^{3/2}}{\mathcal{V}^5}. \tag{4.60}$$

In this case the second term in (4.60) also reproduces the scaling behaviour of the third term in (4.52).

5. Conclusions

The purpose of this article has been to study, as far as possible, the form of loop corrections to the Kähler potential for general Calabi-Yau compactifications and their effect on the scalar potential. The aim has been to extract the parametric dependence on the moduli that control the loop expansion. We have contributed to put the proposed form of leading order string loop corrections on firmer grounds in the sense that they agree with the low-energy effective action behaviour. In particular, it is reassuring that the Coleman-Weinberg formula for the scalar potential fits well with that arising from the BHP conjecture for the corrections to the Kähler potential. Furthermore, the non-contribution of the leading order string loop correction is no longer an accident but it is just a manifestation of the underlying supersymmetry with equal number of bosons and fermions, despite being spontaneously broken.

These results are important for Kähler moduli stabilisation. In particular, even though the string loop corrections to the Kähler potential are subdominant with respect to the leading order α' contribution, they can be more important than non-perturbative superpotential corrections to stabilise non blow-up moduli. The general picture is that all corrections - α' , loop and non-perturbative - play a rôle in a generic Calabi-Yau compactification. We will discuss these matters in more detail in a forthcoming companion article [9].

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A. Survey of moduli stabilisation mechanisms

We have seen that the no-scale structure of the scalar potential will be broken by several contributions which will lead to the following general form

$$V = V_{np} + V_{(\alpha')} + V_{(g_s)}^{KK} + V_{(g_s)}^W + V_{\text{local}} + V_D, \tag{A.1}$$

where V_{np} and $V_{(\alpha')}$ are given by (2.24), and $V_{(g_s)}^{KK}$ and $V_{(g_s)}^W$ are the perturbative contributions from the string loop corrections (3.12) and (3.13). V_{local} is the potential generated by extra local sources and V_D is the usual D-term scalar potential for $N=1$ supergravity

$$V_{(D)} = \frac{1}{2} \left((\text{Re}f)^{-1} \right)^{\alpha\beta} D_\alpha D_\beta, \quad D_\alpha = \left[K_i + \frac{W_i}{W} \right] (T_\alpha)_{ij} \varphi_j. \tag{A.2}$$

We now review moduli stabilisation mechanisms proposed in the literature in order to illustrate the importance of having a deeper understanding of the string loop corrections. From the expression (2.24) we realise that

$$V_{np} \sim e^K (W_{np}^2 + W_0 W_{np}), \quad V_p \sim e^K W_0^2 K_p, \tag{A.3}$$

where in general we have

$$V_p = V_{(\alpha')} + V_{(g_s)}^{KK} + V_{(g_s)}^W, \tag{A.4}$$

for the full perturbative contributions to the scalar potential. Let us explore the possible scenarios which emerge by varying W_0 . As stressed in section 2.2, we can trust the use of solely the leading perturbative corrections to the scalar potential only when the overall volume is stabilised at large values $\mathcal{V} \gg 1$. The first systematic study of the strength of perturbative corrections to the scalar potential was in [23]. Neglecting $V_{(g_s)}^{KK}$, $V_{(g_s)}^W$, V_{local} and $V_{(D)}$, [23] studied the behaviour of the minima of the scalar potential when one varies $|W_0|$. Their results are summarized in the following table:

1) $ W_0 \sim W_{np} \ll 1$	2) $ W_{np} < W_0 < 1$	3) $ W_{np} \ll W_0 \simeq \mathcal{O}(1)$
$ V_{(\alpha')} \ll V_{np} $	$ V_{np} \simeq V_{(\alpha')} $	$ V_{np} \ll V_{(\alpha')} $

1. $|W_0| \sim |W_{np}| \ll 1 \implies |V_{(\alpha')}|/|V_{np}| \sim |\delta K_{(\alpha')}| \sim 1/\mathcal{V} \ll 1 \iff |V_{(\alpha')}| \ll |V_{np}|$

This case is the well-known KKLT scenario [24]. All moduli are stabilised by non-perturbative corrections at an AdS supersymmetric minimum with $D_T W = 0$. A shortcoming of this model is that W_0 must be tuned very small in order to stabilise at large volume and neglect α' or other perturbative corrections. KKLT gave the following fit for the one-parameter case:

$$W_0 = -10^{-4}, \quad A = 1, \quad a \simeq 2\pi/60 \implies \langle \tau \rangle \simeq 113 \iff V \simeq 1.2 \cdot 10^2. \tag{A.5}$$

In addition to $|W_0| \ll 1$, a large rank gauge group (as in SU(60) above) is also necessary to get $a\tau \gg 1$. This is a bit inelegant but a lower rank of the gauge group would imply a much worse fine tuning of W_0 . The authors also proposed a mechanism to uplift the solution to dS, by adding a positive potential generated by the tension of $\overline{D3}$ branes. This represents an explicit breaking within 4D supergravity. Remaining within a supersymmetric effective theory, [25] proposed using D-term uplifting to keep manifest supersymmetry whereas [26] instead proposed F-term uplifting using metastable supersymmetry breaking vacua. Also [27] pointed out that the KKLT procedure in two steps (first the minimisation of S and U_α at tree level and then T_i fixed non-perturbatively) can miss important contributions such as a dS minimum without the need to add any up-lifting term.

We finally notice that this mechanism also relies on the assumption that W_{np} depends explicitly on each Kähler modulus. In the fluxless case, this assumption is very strong as only arithmetic genus 1 cycles [32] would get stringy instanton contributions and $D7$ brane deformation moduli would remain unfixed. The presence of the corresponding extra fermionic zero modes can prevent gaugino condensation and in general could also destroy instanton contributions for non-rigid arithmetic genus 1 cycles. However by turning on fluxes, the $D7$ moduli should be frozen and the arithmetic genus 1 condition can be relaxed. Therefore it is possible that also non-rigid cycles admit nonperturbative effects.

$$2. |W_{np}| < |W_0| < 1 \implies |V_{(\alpha')}|/|V_{np}| \sim |\delta K_{(\alpha')}|/|W_{np}| |W_0| \sim 1 \iff |V_{np}| \simeq |V_{(\alpha')}|$$

[23] pointed out that there is an upper bound on the $|W_0|$ in order to find a KKLT minimum $|W_0| \leq W_{\max}$. W_{\max} is the value of $|W_0|$ for which the leading α' corrections start becoming important and compete with the non-perturbative ones to find a minimum. This minimum will be non-supersymmetric as we can infer from looking at (2.24) which implies that $V \sim \mathcal{O}(1/\mathcal{V}^3)$ at the minimum, while $-3e^K |W|^2 \sim \mathcal{O}(1/\mathcal{V}^2)$. Now since the scalar potential is a continuous function of $|W_0|$, increasing $|W_0|$ from $|W_0| = W_{\max} - \varepsilon$, where we have an AdS supersymmetric minimum, to $|W_0| = W_{\max} + \varepsilon$, will still lead to an AdS minimum which is now non-supersymmetric. Subsequently, when $|W_0|$ is further increased, the α' corrections become more and more important and the minimum rises to Minkowski and then de Sitter and finally disappears. The disappearance corresponds to the α' corrections completely dominating the non-perturbative ones and the scalar potential is just given by the last term in (2.24) that has clearly a runaway behaviour without a minimum.

Unfortunately there is no clear example in the literature that realizes this situation for $\mathcal{V} \gg 1$. In their analysis [23] considered the possibility of getting a Minkowski minimum for the quintic Calabi-Yau $\mathbb{C}P^4_{[1,1,1,1,1]}$ ($\chi = -200$), giving the following fit

$$\begin{aligned} W_0 &= -1.7, \quad A = 1, \quad a = 2\pi/10, \quad \xi = 0.4, \quad \text{Re}(S) = 1 \\ \implies \langle \tau \rangle &\simeq 5 \iff \mathcal{V} \simeq 2. \end{aligned} \tag{A.6}$$

We note that this example, in reality, belongs to the third case since $|W_0| \simeq \mathcal{O}(1)$ where we claimed that no minimum should exist. That is true only for $\mathcal{V} \gg 1$ but in this case $\mathcal{V} \simeq 2$ and the higher α' corrections cannot be neglected anymore. Moreover with $g_s \simeq 1$ the string loop expansion is uncontrolled.

$$3. |W_{np}| \ll |W_0| \sim \mathcal{O}(1) \implies |V_{(\alpha')}|/|V_{np}| \sim |\delta K_{(\alpha')}|/|W_{np}| \gg 1 \iff |V_{(\alpha')}| \gg |V_{np}|$$

This is the more natural situation when $|W_0| \sim \mathcal{O}(1)$. In this case if we ignore the non-perturbative corrections and keep only the α' ones no minimum is present. However there are still $V_{(g_s)}^{KK}$, $V_{(g_s)}^W$, V_{local} and V_D . Thus, let us see two possible scenarios

(a) V_{np} neglected, $V_{(\alpha')} + V_{\text{local}}$ considered

Bobkov [29] considered F-theory compactifications on an elliptically-fibered Calabi-Yau four-fold X with a warped Calabi-Yau three-fold M that admits a conifold singularity at the base of the fibration. Following the procedure proposed by Saltman and Silverstein [28] for flux compactifications on products of Riemann surfaces, he added n_{D7} additional pairs of $D7/\overline{D7}$ -branes and n_7 extra pairs of (p, q) $7/\overline{7}$ -branes wrapped around the 4-cycles in M placed at the loci where the fiber T^2 degenerates. These extra local sources generate positive tension and an anomalous negative $D3$ -brane tension contribution to V_{local} which,

in units of $(\alpha')^3$, reads

$$V = -\chi (2\pi)^{13} N_{\text{flux}}^2 \left(\frac{g_s^4}{\mathcal{V}_s^3} \right) - N_7 \left(\frac{g_s^3}{\mathcal{V}_s^2} \right) + n_7 \left(\frac{g_s^2}{\mathcal{V}_s^{4/3}} \right) + n_{D7} \left(\frac{g_s^3}{\mathcal{V}_s^{4/3}} \right), \quad (\text{A.7})$$

where \mathcal{V}_s is the string frame volume and $N_7 = (n_{D7}^3 + n_7^3)$ is an effective parameter given in terms of triple intersections of branes. By varying the various parameters, this is argued to give a discretuum of large-volume non-supersymmetric AdS, Minkowski and metastable dS vacua for Calabi-Yau threefolds with $h_{1,1} = 1$ (this implies $\chi < 0$). The fit proposed is for the dS solution:

$$|W_0| \simeq (2\pi)^2 N_{\text{flux}} > 1, \quad \chi = -4, \quad N_{\text{flux}} = 3, \quad n_7 = 1, \quad n_{D7} = 73, \\ g_s \simeq 5 \cdot 10^{-3} \implies \mathcal{V} \simeq 3 \cdot 10^4. \quad (\text{A.8})$$

The integer parameters are tuned to obtain a pretty small g_s so that the effect of string loop corrections can be safely neglected. In this scenario, in which supersymmetry is broken at the Kaluza-Klein scale, the stabilisation procedure depends on local issues, while we would prefer to have a more general framework where we could maintain global control.

(b) V_{np} neglected, $V_{(\alpha')} + V_{(g_s)}^{KK} + V_{(g_s)}^W$ considered

Berg, Haack and Körs [5], following their exact calculation of the loop corrections for the $N=1$ toroidal orientifold $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ analyzed if these corrections could compete with the α' ones to generate a minimum for V . By treating the three toroidal Kähler moduli in $T^6 = T^2 \times T^2 \times T^2$ on an equal footing they reduce the problem to a 1-dimensional one. They neglect $V_{g_s}^{KK}$ as it is suppressed by higher powers of the dilaton and compare just $V_{\alpha'}$ and $V_{g_s}^W$. The schematic form of the scalar potential is

$$V \sim \frac{\xi |W_0|^2}{\mathcal{V}^3} + \frac{\delta}{\mathcal{V}^{10/3}}. \quad (\text{A.9})$$

It turns out that $\delta > 0$, and so as $\xi \sim -\chi$, they need a positive Euler number $\chi > 0$ in order to find a minimum, while the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ toroidal example has a negative Euler number. They instead consider the $N=1$ toroidal orientifold T^6/\mathbb{Z}_6 that satisfies the condition $\chi > 0$. A non-supersymmetric AdS minimum is now present but as the loop corrections are naturally subleading with respect to the α' ones, they must fine tune the complex structure moduli to get large volume. They find

$$|W_0| \sim \mathcal{O}(1), \quad \text{Re}(U) \simeq 650, \quad \text{Re}(S) = 10 \\ \implies \langle \tau \rangle \simeq 10^2 \iff \mathcal{V} \simeq 10^3. \quad (\text{A.10})$$

The fine-tuning comes from assuming the complex structure moduli are stabilised at large values. A similar scenario has been studied also by von Gersdorff

and Hebecker [14]. In addition, Parameswaran and Westphal [30] studied the possibility to have a consistent D-term uplifting to de Sitter in this scenario.

4. We have assumed above that when $|W_0| \sim \mathcal{O}(1)$ perturbative corrections always dominate non-perturbative ones, which can therefore be neglected. But is this naturally always the case? In order to answer this question, let us now consider scenarios in which V_{np} and $V_{(\alpha')}$ compete while $|W_0| \sim \mathcal{O}(1)$.

- (a) $V_{(g_s)^{KK}} + V_{(g_s)^W}$ neglected, $V_{np} + V_{(\alpha')}$ considered \implies large volume

This situation was studied by Westphal [31] following the work of Balasubramanian and Berglund, finding a dS minimum at large volume for the quintic. However this result extends to other Calabi-Yau three-folds with just one Kähler modulus. He presents the following fit

$$\begin{aligned} W_0 &= -1.7, \quad A = 1, \quad a = 2\pi/100, \quad \xi = 79.8, \quad \text{Re}(S) = 1 \\ \implies \quad \langle \tau \rangle &\simeq 52 \iff \mathcal{V} \simeq 376. \end{aligned} \tag{A.11}$$

The non-perturbative corrections are rendered important by using a large-rank gauge group $SU(100)$ for gaugino condensation. This is not fine-tuned but is contrived. The loop corrections, which may be important, are not considered here.

- (b) $V_{(g_s)^{KK}} + V_{(g_s)^W}$ neglected, $V_{np} + V_{(\alpha')}$ considered \implies exponentially large volume

This situation is appealing since it provides a positive answer to our basic question. Balasubramanian, Berglund and two of the present authors [8] developed these scenarios which now go under the name of Large Volume Models, which is a bit misleading as large volume is always necessary to trust a solution. They should be more correctly called LARGE Volume Models because the volume is exponentially large. In this framework, both non-perturbative and α' corrections compete naturally to get a non-supersymmetric AdS minimum of the scalar potential at exponentially large volume. This is possible by considering more than one Kähler modulus and taking a well-defined large volume limit. For one modulus models, the work of [23] and [31] shows that with the rank of the gauge group $SU(N)$ in the natural range $N \simeq 1 \div 10$, it is impossible to have a minimum.

However, if we have more generally $h_{1,1} > 1$, this turns out to be possible. The simplest example of such models is for the hypersurface $\mathbb{C}P^4_{[1,1,1,6,9]}$. The overall volume in terms of 2-cycle volumes is given by

$$\mathcal{V} = \frac{1}{6} (3t_1^2 t_5 + 18t_1 t_5^2 + 36t_5^3), \tag{A.12}$$

and the 4-cycle volumes take the form

$$\tau_4 = \frac{t_1^2}{2}, \quad \tau_5 = \frac{(t_1 + 6t_5)^2}{2}, \tag{A.13}$$

for which it is straightforward to see that

$$\mathcal{V} = \frac{1}{9\sqrt{2}} \left(\tau_5^{3/2} - \tau_4^{3/2} \right). \tag{A.14}$$

The reason why τ_4 and τ_5 are considered instead of τ_1 and τ_5 as outlined in section 2.1, is that these are the only 4-cycles which get instanton contributions to W when fluxes are turned off [33]. As we will describe in our companion paper [9], to get LARGE Volume Models, we require that W_{np} depends only on blow-up modes which resolve point-like singularities, as τ_4 in this case. Such cycles are always rigid cycles and thus naturally admit nonperturbative effects. If we now take the large volume limit in the following way

$$\begin{cases} \tau_4 \text{ small,} \\ \tau_5 \gg 1, \end{cases} \tag{A.15}$$

the scalar potential looks like

$$V = V_{np} + V_{(\alpha')} \sim \frac{\lambda\sqrt{\tau_4}e^{-2a_4\tau_4}}{\mathcal{V}} - \frac{\mu\tau_4e^{-a_4\tau_4}}{\mathcal{V}^2} + \frac{\nu}{\mathcal{V}^3}, \quad \lambda, \mu, \nu \text{ constants} \tag{A.16}$$

with a non-supersymmetric AdS minimum located at

$$\tau_4 \sim (4\xi)^{2/3} \quad \text{and} \quad \mathcal{V} \sim \frac{\xi^{1/3} |W_0|}{a_4 A_4} e^{a_4 \tau_4}. \tag{A.17}$$

The result that we have found confirms the consistency of our initial assumption (A.15) in taking the large volume limit. Inserting in following natural choice of parameters, we find

$$\begin{aligned} W_0 &= 1, \quad A_4 = 1, \quad a_4 = 2\pi/7, \quad \xi = 1.31, \quad \text{Re}(S) = 10 \\ \implies \quad \langle \tau_4 \rangle &\simeq 41 \iff \mathcal{V} \simeq 3.75 \cdot 10^{15}. \end{aligned} \tag{A.18}$$

Therefore τ_4 is stabilised small whereas $\tau_5 \gg 1$, and the volume can be approximated as

$$\mathcal{V} \sim \tau_5^{3/2}, \tag{A.19}$$

and

$$\tau_4 \sim t_1^2, \quad \tau_5 \sim t_5^2. \tag{A.20}$$

Looking at (A.17) we can realise why in this case we are able to make V_{np} and $V_{(\alpha')}$ compete naturally. In fact, in general $V_{(\alpha')} \sim 1/\mathcal{V}^3$ and $V_{np} \sim e^{-a_4\tau_4}/\mathcal{V}^2$, but (A.17) implies $V_{np} \sim 1/\mathcal{V}^3 \sim V_{(\alpha')}$. The non-perturbative corrections in the big modulus τ_5 will be, as usual, subleading. An attractive feature of these models is that they provide a method of generating hierarchies. In fact the result (A.18), for $M_P \sim 2.4 \cdot 10^{18}$ GeV, produces an intermediate string scale

$$M_s \simeq \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{11} \text{ GeV}, \tag{A.21}$$

and this can naturally give rise to the weak scale through TeV-scale supersymmetry

$$M_{\text{soft}} \sim m_{3/2} = e^{K/2} |W| \sim \frac{M_P}{\mathcal{V}} \sim 30 \text{ TeV}. \quad (\text{A.22})$$

This setup naturally fixes all the moduli while generating hierarchies. However, it ignores further perturbative corrections as the g_s ones. It is thus crucial to check if they do not destroy the picture. Berg, Haack and Pajer applied their guess (3.12) to derive these string loop corrections to the Kähler potential. From (3.12) it is straightforward to get⁷

$$\delta K_{(g_s)}^{KKK} \sim \frac{\mathcal{C}_4^{KK} \sqrt{\tau_4}}{\text{Re}(S) \mathcal{V}} + \frac{\mathcal{C}_5^{KK} \sqrt{\tau_5}}{\text{Re}(S) \mathcal{V}}, \quad (\text{A.23})$$

$$\delta K_{(g_s)}^W \sim \frac{\mathcal{C}_4^W}{\text{Re}(S) \sqrt{\tau_4} \mathcal{V}} + \frac{\mathcal{C}_5^W}{\text{Re}(S) \sqrt{\tau_5} \mathcal{V}}. \quad (\text{A.24})$$

The corrections (A.23) turn out to yield subleading corrections to the scalar potential of the form

$$V_{(g_s)}^{KKK} \sim \frac{(\mathcal{C}_4^{KK})^2 W_0^2}{\text{Re}(S)^2 \mathcal{V}^3 \sqrt{\tau_4}} + O(\mathcal{V}^{-10/3}), \quad (\text{A.25})$$

even if one tries to fine tune the coefficients \mathcal{C}_4^{KK} pretty large, $\mathcal{C}_4^{KK} \simeq 20 \div 40$. We therefore conclude that the LARGE Volume Scenario is safe.

This survey of moduli stabilisation mechanisms has shown that a deeper understanding of string loop corrections to the Kähler potential in Calabi-Yau backgrounds is highly desirable. In KKLТ stabilisation, the magnitude of the perturbative corrections is what determines the regime of validity of the stabilisation method. In all other methods of stabilisation, perturbative corrections enter crucially into the stabilisation procedure, and so not only α' but also g_s corrections should be taken into account.

These loop corrections are neglected in the cases (3a), (4a) and (4b), but we learnt from the case (3b) that they can change the vacuum structure of the system studied. However in this situation a significant amount of fine tuning was needed to make them compete with the α' corrections to produce a minimum at large volume. In case (4b), the loop corrections did not substantially affect the vacuum structure unless they were fine-tuned large. Therefore one would tend to conclude that these string loop corrections will in general be subdominant and so that it is safe to neglect them.

While this may be true for models with relatively few moduli, we will see in [9] that loop corrections can still play a very important rôle in moduli stabilisation, in particular lifting flat directions in LARGE Volume Models. In this case the fact that they are subdominant will turn out to be a good property of these corrections since they can lift flat directions without destroying the minimum already found in the other directions of the Kähler moduli space.

⁷We note that in this case, as argued by Curio and Spillner [16], $\delta K_{(g_s)}^W$ is absent, because in $\mathbb{C}P_{[1,1,1,6,9]}^4$ there is no intersection of the divisors that give rise to nonperturbative superpotentials if wrapped by D7 branes.

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